17th INTERNATIONAL GEOMETRY SYMPOSIUM

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ABSTRACTS BOOK

ERZİNCAN BİNALİ YILDIRIM ÜNİVERSİTESİ
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17TH INTERNATIONAL GEOMETRY SYMPOSIUM

ABSTRACTS BOOK

Proceedings of the 17th International Geometry Symposium

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FOREWORD

Hosted by Erzincan Binali Yıldırım University between June 19-22, 2019, the 17th International Geometry Symposium was held in Erzincan which is a beautiful and has historical background in the east of Turkey. Undergraduate students aiming to do scholarly studies as well as new researchers had a great opportunity of getting together with highly experienced researchers. In light of scientific developments in Geometry, presentations were made, and discussions were held, thus paving the way for new research. All the studies in this booklet were peer-reviewed, and then brought up to the attention of the audience. Through their presentations, the keynote speakers helped the researchers explore some new ways of thinking.

In making our event happen, special thanks go to the following: Office of the Rector of Erzincan Binali Yıldırım University for letting us use its facilities, office of the Governor of Erzincan, Erzincan Municipality, Erzincan Culture and Education Foundation and all our collagues and students who worked with us to make this symposium a success.

Assoc. Prof. Dr. Sezai KIZILTUĞ

Erzincan Binali Yıldırım University

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Invited Speakers

The Gifts that God Gave Us in terms of Mathematics

H. Hilmi HACISALIHOGLU

Abstract

In this study, the gifts that God gave us in terms of mathematics have presented. The most important guide in the universe is mathematics. Mathematics developed with the universe. In fact, mathematics existed before the universe, and will also exist after the universe. There is also the language of the universe. And this language is mathematics itself. This language also has the alphabet. There is a model that is important for the formation of the universe. This model is similarity. The basis of similarity is symmetry. There is no place for chance in the formation of the universe. Because mathematics was used in this formation.

Rotation Minimizing Vector Fields and Applications

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Abstract

In three dimensional space, there are important applications of Bishop frame as "non-rotating frame". In n-dimensional space, generalization of this frame is called a Rotation minimizing frame. In this talk, applications of Rotation minimizing vector fields, in n-dimensional space and n-dimensional Minkowski space, will be given. With the help of a Rotation minimizing frame characterizations of rectifying and spherical curves and also helices will be presented.

Keywords: Bishop Frame; Rotation Minimizing Frame; Rectifying Curve.

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Magnetic Curves and Generalizations

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Abstract

Geodesics on a Riemannian manifold (M, g) are given by a second order nonlinear differential equation: the *Euler-Lagrange equation of motions*, locally expressed as: $\ddot{x}^k(s) + \Gamma_{ij}^k(x(s))\dot{x}^i(s)\dot{x}^j(s) = 0$, obtained as critical point of the *kinetic energy* (also called the *action integral*)

$$E(\gamma) = \int \frac{1}{2}|\gamma'(s)|^2.$$

Let now ω be a 1-form called the *potential 1-form*. For a smooth curve $\gamma: [a, b] \longrightarrow M$ we consider the functional

$$LH(\gamma) = \int_a^b \left(\frac{1}{2}\langle\gamma'(t), \gamma'(t)\rangle + \omega(\gamma'(t))\right) dt,$$

often called the *Landau Hall functional* for the curve γ, which is a perturbation of the kinetic energy of the curve with the potential ω. The critical points of the LH functional are solutions of the equation $\frac{d}{d\epsilon} LH(\gamma_\epsilon)|_{\epsilon=0} = 0$, that is

$$\frac{d}{d\epsilon} LH(\gamma_\epsilon)|_{\epsilon=0} = -\int_a^b g(\nabla_{\gamma'}\gamma' - \phi(\gamma'), V)dt = 0,$$

which is equivalent to

$$\nabla_{\gamma'}\gamma' - \phi(\gamma') = 0.$$

Here ϕ is a (1,1) tensor field on M, called the *Lorentz force* and defined by $g(\phi X, Y) = d\omega(X, Y)$, for all X, Y tangent to M. One can consider a weaker condition for ϕ, that is it is obtained from a closed 2-form usually called a *magnetic field* on the manifold. Moreover, if we remove also this condition and consider only that ϕ is skew-symmetric, one gets *trajectories* on manifolds.

The notion of geodesic is generalized to maps between Riemannian manifolds. A map $f: (N, h) \to (M, g)$ between Riemannian manifolds is said to be *harmonic* if it is a critical point of the energy functional:

$$E(f) = \int_N \frac{1}{2}|df|^2 dv_h$$

under compactly supported variations. The Euler-Lagrange equation of this variational problem is given by the vanishing of the *tension field of f*, that is

$$\tau(f) = \text{div} = 0.$$

The Landau Hall functional for maps. Let $f: N \longrightarrow M$ be a smooth maps between two Riemannian manifolds. Let ξ be a global vector field on N and ω be a 1-form on M. Let us define the following functional for f associated to ξ and ω:

$$LH(f) = E(f) + \int_N \omega(df(\xi))dv_h.$$

Definition. The map f is called *magnetic* with respect to ξ and ω if it is a critical point of the Landau Hall integral defined above, i.e. the first variation $\frac{d}{d\epsilon} LH(f_\epsilon)|_{\epsilon=0}$ is zero.

Theorem. [Inoguchi & Munteanu, 2014] Let $f:(N,h) \longrightarrow (M,g)$ be a magnetic map with respect to ξ and ω. Then f satisfies the Lorentz equation

$$\tau(f) = \phi(f_*\xi).$$

As before, we can replace the exactness of the 2-form by the closedness.

This notion generalizes both magnetic curves and harmonic maps. It helps us also to define new notions such as magnetic vector fields, magnetic endomorphisms on the tangent bundle, magnetic submanifolds and many other.

Keywords: Magnetic Curves.

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Holomorphic Manifolds Over Algebras and Their Applications

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Abstract

In first part of our presentation we give the fundamental results and some theorems concerning geometry of holomorphic hypercomplex manifolds which will be needed for the later treatment of special types of hypercomplex manifolds. Let now M_n be a differentiable manifold and $T(M_n)$ its tangent bundle. Two types of lift problems have been studied in the previous works: a) The lift of structures (functions, vector fields, forms, tensor fields, linear connections, etc.) from the base manifold to the tangent bundle; b) The definition of geometric structures on the total manifold $T(M_n)$, by means of a specific geometric structure on M_n or on the fibre bundle $T(M_n)$. In the present working we continue such a study by considering the structure given by the dual numbers on the tangent bundle and defining new lifts of functions, vector fields, forms, tensor fields and linear connections. Also, we investigate the complete lift ${}^C\varphi_{T^*M}$ of almost complex structure φ to cotangent bundle and prove that it is a transform by symplectic-musical isomorphism $\omega^{\flat}$ of complete lift ${}^C\varphi_{TM}$ to tangent bundle if the triple (M,ω,φ) is an almost holomorphic A-manifold [1].

Keywords: Hypercomplex algebra; Bundle; Lift.

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Cubic Surfaces Over Small Finite Fields

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Abstract

In the 19th century, cubic surfaces have been studied over the real and complex numbers. Starting with Fermat in the 17th century, the work of Cayley, Salmon, Clebsch, Schlaefli, Klein and Hilbert laid the foundation of modern algebraic geometry. The geometry of these surfaces is very interesting, for instance it can be shown that each smooth surface has exactly 27 lines. In this talk, we will consider cubic surfaces over finite fields. The properties of surfaces over finite fields are very much the same as over infinite fields. However, because of finiteness, we can use computers to classify surfaces with 27 lines exhaustively, at least over small fields. We describe our work [1], which led to the classification of all cubic surfaces with 27 lines over fields whose order is less than one hundred. Interesting examples of surfaces arise. We are especially interested in the case of characteristic two, where Hirschfeld's family [3] exists. We find three new families, different from the known family of Hirschfeld. Over fields of odd order, we encounter the Hilbert, Cohn-Vossen surface [4]. Geometric invariants like the number of Eckardt points [2] are able to tell these surfaces apart.

Keywords: Cubic Surfaces, Finite Field, Classification.

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The Ethical Principles for Scientific Research and Publications

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Abstract

In scientific research and publications, all the principals and rules that the researcher should take into consideration are called scientific ethics. In this study the master's and doctoral thesis produced in the universities and the ethical violations in the preparation of the articles produced from these thesis are discussed. Finally, it is mentioned that, the young researchers should be aware of what they should pay attention not to be exposed to these ethical violations.

Keywords: Scientific ethics, Ethical violations.

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Parallel Second Order Tensors on Vaisman Manifolds

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Abstract

In this paper, we study the class of parallel tensor fields α of (0,2)-type in a Vaisman geometry (M,J,g) and give a sufficient condition for the reduction of such symmetric tensors α to a constant multiple of g is given by the skew-symmetry of α with respect to the complex structure J.

Keywords: Vaisman manifold; Lee vector field; Ricci soliton

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Generalized Paracontact Metric Manifolds

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Abstract

Two different notions of almost paracontact structures (which are compatible or anti-compatible with the metric), well known in literature, are united and generalized here. Several formulas of paraholomorphic maps are established and a result of Lichnerowicz type is obtained. The connection transformations which have the same system of paracontact-planar Legendre curves are characterized. Conformal changes of metrics which preserve geodesics (resp. paracontact-planar Legendre curves) are studied.

Keywords: Paracontact metric manifolds; Legendre curves

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Mathematical Reasoning in Teaching Science, Education and Misconceptions

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Abstract

As known some findings of recent years show that teaching and education are not at the desired level in science and mathematics, even relatvely declined. In this presentation we will focus on some basic problems of education and teaching of science and mathematics. We will also talk about the misconceptions that can lead to lifelong chain of errors in education and training. For instance in mathematics, if the limit which is the basic concept of the the analysis, is understood 'as substituting the given value' then this leads to misunderstanding of the following concepts in all sciences. For this reason, the definition of any concept should be given fully and clearly

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Abstracts of Oral Presentations

A New Class of Curves Generalizing Helix and Rectifying Curves

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Abstract

We introduce a new class of curves α called the f-rectifying curves, which its f-position vector defined by $\alpha_f = \int f(s)d\alpha$ always lie in its rectifying plane, where f is an integrable function in arclength s of α. The class f-rectifying curves generalize helix and rectifying curves for some particular cases of the functions f, f = 0 and f is a constant respectively. The classification and the characterization of such curves in terms of their curvature and the torsion functions are given and an example is presented.

Key words: f-rectifying; f-position vector; helix; rectifying.
AMS Subject Classification: 53A04, 53A17

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The Generalized Taxicab Distance Formulae

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Abstract

In this talk, we first determine the generalized taxicab distance formulae between a point and a line and two parallel lines in the real plane, then we determine the generalized taxicab distance formulae between a point and a plane, two parallel planes, a point and a line, two parallel lines and two skew lines in three dimensional space, giving also the relations between these formulae and their well-known Euclidean analogs. Finally, we give the generalized taxicab distance formulae between a point and a plane, a point and a line and two skew lines in *n*-dimensional space, by generalizing the concepts used for three dimensional space to *n*-dimen-sional space.

Keywords: Generalized taxicab distance; metric; taxicab geometry; three dimensional space; *n*-dimensional space.

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A Study on Lightlike Submanifolds of Golden Semi-Riemannian Manifolds

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Abstract

We introduce lightlike submanifolds of golden semi-Riemannian manifolds. Particularly, we study semi-invariant lightlike submanifolds of golden semi-Riemannian manifolds. We find some conditions for integrability of distributions of such submanifolds and investigate the geometry of leaves of distributions. Moreover, we study totally umbilical lightlike submanifolds of golden semi-Riemannian manifolds and give an example.
Keywords: Golden semi-Riemannian manifolds; Golden structure; Lightlike submanifolds; Semi-invariant lightlike submanifolds.

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A Study on Some Special Riemannian Manifolds with Semi-Symmetric Metric Connection

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Abstract

In this study, we examine special vector fields on some Riemannian manifolds admitting a semi-symmetric metric connection. We consider φ(Ric)-vector fields, parallel vector fields and torqued vector fields on these manifolds and prove some theorems related to these vector fields, for instance, such a manifold with constant curvature is conformally flat if it admits a φ(Ric)-vector field and finally, we give an example.

Keywords: Semi-symmetric metric connection; Vector fields; Special Riemannian manifolds.

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Some Results on Weak M-Projective Symmetric Sasakian Manifolds

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Abstract

In this paper, we consider the M-projective curvature tensor on Sasakian manifolds. We have defined weakly M-projective symmetric and weakly M-projective Ricci symmetric Sasakian manifolds and obtained some results. Finally, we investigate the case where weakly M-projective Ricci symmetric manifolds have the M-projective Ricci tensor W^* to be cyclic, and we have expressed some theorems.

Keywords: Sasakian manifold; M-projective curvature tensor; Weakly M-projective symmetric Sasakian manifold; Weakly M-projective Ricci symmetric Sasakian manifold.

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Some Notes on Projectable Linear Connection

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Abstract

Using the fiber bundle M over a manifold B, we define a semi-tangent (pull-back) bundle tB. We *analysis* the complete and horizontal lifts of projectable linear connection for semi-tangent (pull-back) bundle tB. In addition, a new example for good square is presented in this work.

Keywords: Vector field, complete lift, projectable linear connection, pull-back bundle, semi-tangent bundle.

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On Quasi-Para-Sasakian Manifolds

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Abstract

Basic structure and curvature identities of quasi-para-Sasakian manifolds are given. Also, quasi-para-Sasakian manifolds of constant curvature are completely characterized. An example of 3-dimensional proper quasi-para-Sasakian manifold which is neither the paracosymplectic manifold nor the para-Sasakian manifold is presented. Then, a characterization of three-dimensional conformally flat quasi-para-Sasakian manifold is given.

Keywords: quasi-para-Sasakian manifold; conformally flat; constant curvature.

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Notes On Constant Precession Curve

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Abstract

In this paper we determine the geodesic curvature and geodesic torsion of constant precession curve, and the normal curvature of the circular hyperboloid of one-sheet, in the direction of tangent vector of the constant precession curve, by the meanings of the Darboux frame of the curve. We give the causal character of constant precession curve in Minkowski space and we state the constant angle that its principal normal makes with fixed direction. Moreover, we give some angles just as, the angle between the osculating plane of the constant precession curve and the tangent plane to the circular hyperboloid of one-sheet; the angle between principal unit normal of constant precession curve and unit normal vector of circular hyperboloidof one-sheet, in terms of curvatures of the curve.

Keywords: Constant precession curve.

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Hamiltonian Mechanical Systems with respect to the Lifts of Almost Product Structure on Cotangent Bundle

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Abstract

The differential geometry and mathematical physics has lots of applications. The Hamiltonian mechanical systems are very important tools for differential geometry, classical and analytical mechanics. There are many studies about Hamiltonian mechanical systems, formalisms and equations. Because of the investigation of tensorial structures on manifolds and extension by using the lifts to the tangent or cotangent bundle, it is possible to generalize to differentiable structures on any space (resp. manifold) to extended spaces (resp. extended manifolds) (Sasaki 1958, Salimov 2013, Yano and Ishihara 1973). In this study, the mathematical models of the Hamiltonian mechanical systems are structured on the horizontal and the vertical lifts of an almost product structure in cotangent bundle. In the end, the geometrical and physical results related to Hamiltonian mechanical systems are concluded. In this context this paper consists of two main sections. In the first section, we give some properties about the horizontal, complete and vertical lifts of vector and covector fields on the cotangent bundle. Later, we will give some general information about the Hamiltonian equations and mechanical systems. In final section, the results of the Hamiltonian equations with respect to horizontal and vertical lifts of an almost complex structure and the Hamiltonian mechanical systems will be investigated on cotangent bundle $T^*(M)$.

Keywords: Hamiltonian mechanical systems; Lifts; Almost product structure; Cotangent bundle.

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The Transformation of the Evolute Curves using by Lifts on R^3 to Tangent Space TR^3

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Abstract

"How we can speak about the features of evolute curve on space TR^3 by looking at the characteristics of the first curve α?" In this paper, we investigate the answer of this question using by lifts. In this direction firstly, we define the evolute curve of any curve with respect to the vertical, complete and horizontal lifts on space R^3 to its tangent space $TR^3=R^6$. Secondly, we examine the Frenet-Serret aparatus $\{T^*(s),N^*(s),B^*(s),\kappa^*,\tau^*\}$ and the Darboux vector W^* of the evolute curve α^* according to the vertical, complete and horizontal lifts on TR^3 by depend on the lifting of Frenet-Serret aparatus $\{T(s),N(s),B(s),\kappa,\tau\}$ of the first curve α on space R^3. In addition, we include all special cases the curvature $\kappa^*(s)$ and torsion $\tau^*(s)$ of the Frenet-Serret aparatus $\{T^*(s),N^*(s),B^*(s),\kappa^*,\tau^*\}$ of the evolute curve α^* with respect to the vertical, complete and horizontal lifts on space R^3 to its tangent space TR^3. As a result of this transformation on space R^3 to its tangent space TR^3, we could have some information about the features of evolute curve of any curve on space TR^3 by looking at the characteristics of the first curve α. Moreover, we get the transformation of the evolute curves using by lifts on R^3 to tangent space TR^3. Finally, some examples are given for each curve transformation to validated our theorical claims.

Keywords: Vector fields; Evolute curve; Vertical lift; Complete lift; Horizontal lift; Tangent space.

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On Minimal Surfaces in Galilean Space

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Abstract

In this paper, we investigated the minimal surfaces in three dimensional Galilean space $\mathbb{G}^3$. We showed that the condition of minimality of a surface area is locally equivalent to the mean curvature vector H vanishes identically. Then, we derived the necessary and sufficient conditions that the minimal surfaces have to satisfy in Galilean space.

Keywords: Minimal surfaces; Area of a surface; Galilean space.

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Some Remarks for a New Metric in the Cotangent Bundle

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Abstract

In this paper, we study a new metric $\breve{G} = {}^{R}\nabla + \sum_{i,j=1}^{m} a^{ji}\delta p_j \delta p_i$ in the cotangent bundle, where ${}^{R}\nabla$ is the Riemannian extension and a^{ji} is a symmetric (2,0)-tensor field on a differentiable manifold. Then we investigate the holomorphy property of the metric $\breve{G}$ by using compatible almost complex structure in the cotangent bundle.

Keywords: Cotangent bundle; Riemannian extension; Almost complex structure.

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Warped Product Submersions

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Abstract

Warped product manifolds plays very important roles to construct cosmological models in general relativity theory. For instance Schwarzschild and Robertson-Walker cosmolgical models are well known examples of warped product manifold [1]. It is well known that the notion of warped product manifolds appeared in the differential geometry as a generaliztion of the Riemannian product manifolds [2].

Nash embeding theorem which was given by J.F.Nash state that every Riemann manifold can be isometrically immersed in some Euclidean spaces with.sufficently high dimensions [3]. Due to the Nash's theorem, one can say that every warped product $M_0 \times_{\rho_1} M_1$ manifold can be embedded to some Euclidean spaces.

In view of Nash's theorem, the following decomposition theorem of S.Nölker is known as a generalization of J.Moore's Theorem.

Theorem [4]: Let ϕ : $N_0 \times_{\rho_1} N_1 \times_{\rho_2} N_2 \times \ldots \times_{\rho_k} N_k \rightarrow R^n(c)$ be an isometric immersion into a Riemannian manifold of constant curvature c. If h is the second fundamental form of ϕ and h(X_i, X_j)=0, for all vector fields X_i and X_j tangent to N_i and N_j respectively, with i≠j, then, locally, ϕ is a warped product immersion.

Recently B.Y.Chen studied fundamental geometric properties of warped product immersions and collected these results in extensive and comprehensive survey of warped product manifolds and submanifolds [5].

Naturally one can ask if $\phi : M_1 \times_f M_2 \rightarrow R^n(c)$ is a Riemannian submersion then are there exist a warped product representation $R^n(c)$ of $M_1 \times_f M_2$ such that

$\phi = \phi_1 \times \phi_2 : M_1 \times_f M_2 \rightarrow N_1 \times_\rho N_2$

given by /$\phi_1 \times \phi_2$)(p_1,p_2)=($\phi_1(p_1)$, $\phi_2(p_2)$) is a Riemannian submersion ?

In this study we defined warped product submersion and give some examples and also studied basic properties of such as a submersion. and arrived following theorem:

Theorem Let $\phi = \phi_1 \times \phi_2 : M_1 \times_f M_2 \rightarrow N_1 \times_\rho N_2$ be a warped product submersion between two product manifolds. Then we have

i)The warping function f is constant on fibers of ϕ_1,

ii) ϕ is mixed totally geodesic,

iii) The squared norm of the second fundamental form of fibers of ϕ satisfies

$\|T\|^2 \geq (m_2-n_2)\|H(\mathrm{grad} f)\|^2$

with the equality holding if and only if ϕ_1 and ϕ_2 have totally geodesic fibers.

Keywords: Isometric immersion , Rieamannian submersion, warped product,

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On Developable Ruled Surfaces in Pseudo-Galilean Space

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Abstract

In this paper, we investigated the ruled surfaces in the three-dimensional pseudo-Galilean space. We obtained the distribution parameter of the ruled surface by using the Frenet frame of directrix curve. Moreover, we derived the necessary conditions to construct a developable ruled surface in the pseudo-Galilean space.

Keywords: Ruled surfaces, pseudo-Galilean space, Developable

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Generalized Paracontact Metric Manifolds

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Abstract

Two different notions of almost paracontact structures (which are compatible or anti-compatible with the metric), well known in literature, are united and generalized here. Several formulas of paraholomorphic maps are established and a result of Lichnerowicz type is obtained. The connection transformations which have the same system of paracontact-planar Legendre curves are characterized. Conformal changes of metrics which preserve geodesics (resp. paracontact-planar Legendre curves) are studied.

Keywords: Paracontact metric manifolds; Legendre curves

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Parallel Second Order Tensors on Vaisman Manifolds

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Abstract

In this paper, we study the class of parallel tensor fields α of (0,2)-type in a Vaisman geometry (M,J,g) and give a sufficient condition for the reduction of such symmetric tensors α to a constant multiple of g is given by the skew-symmetry of α with respect to the complex structure J.

Keywords: Vaisman manifold; Lee vector field; Ricci soliton

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Smarandache Curves according to the Sabban Frame belong to Spherical Indicatrix Curve of the Salkowski Curve

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Abstract

In this study, Smarandache curves were defined according to the Sabban frame belong to spherical indicatrix curve of the Salkowski curve. Then of these curves were calculated geodesic curvatures. Each curve is drawn with maple program.

Keywords: *Salkowski curve; Smarandache curve; Sabban frame*

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Some Properties of Riemannian Submersions Between Ricci Solitons

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Abstract

In this paper, a Riemannian submersion from Riemannian manifold admitting a Ricci soliton is studied. Here, some characterizations about the vertical and horizontal distributions of such a submersion are given. Also, necessary and sufficient conditions for any fiber of Riemannian submersion from Ricci soliton to be totally geodesic or totally umbilical are obtained.

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Keywords: Riemannian submersion; Ricci soliton.

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Biharmonic Legendre Frenet Curves on Generalized Indefinite Sasakian Space Forms

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Abstract

Biharmonic Frenet Legendre curves are discussed on generalized indefinite Sasakian space form. The warping product function of warped product of the real number set and a generalized indefinite complex space form containing biharmonic curves is computed.

Keywords: Biharmonic Map; Legendre Curve; Indefinite Sasakian Space Form.

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On Minimal Complex Lightlike Hypersurfaces

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Abstract

In this paper, minimal complex lightlike hypersurface of an indefinite Kaehler manifold is investigated. Some results on strongly minimal Monge-type lightlike hypersurfaces of 4-dimensional complex Euclidean space are given.

Keywords: Curvature; Bochner Kaehler Manifold.

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A Characterization of the De Sitter Space

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Abstract

In this paper, we characterize the de Sitter space by means of spacelike and timelike curves that fully lies on it. For this purpose, we consider the tangential part of the second derivative of the unit speed curve on the hypersurface, and obtain the vector equations of the geodesics. We find the geodesics as hyperbolas, ellipses, and helices. Moreover, we give an example of null curve with constant curvature in 4−dimensional Minkowski space and we illustrate the geodesics of S1_1 (r) × R

Keywords: de Sitter space, geodesic, curve with constant curvature, Lorentz-Minkowski space.

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Spherical Curves in Finsler 3-Space

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Abstract

In this work, we investigate the general characteristics of the Finslerian spherical curves in Finsler 3-space. We obtain some characterizations for these curves. Moreover, we give various examples and visualized their images on the Randers sphere.

Keywords: Special curves; Finsler space; Frame fields.

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Special Helices on the Ellipsoid

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Abstract

In this study, we investigate a curve whose position vector field makes a constant angle with the constant vector field on the Ellipsoid S_E^2. We call this curve is a special helix. Then, we obtain the parametric representation of all special helices on the ellipsoid S_E^2. Moreover, we present various examples and plotted their images.

Keywords: Special curves; Euclidean space; Frame fields.

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Notes about the $g-lift$ of Affine Connection

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Abstract

In this study, the $g-lift$ of the affine connection are determined on the cotangent bundle via the musical isomorphism and the $g-lift$ of the curvature tensor of the affine connection are obtained with the same method.

Keywords: g-lift; Complete lift; Connection; Curvature tensor; Musical isomorphism; Cotangent bundle.

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$\boldsymbol{\beta}$ −Kenmotsu Lorentzian Finsler Manifolds

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Abstract

The purpose of this study is to introduce some properties and results for β −Kenmotsu Lorentzian Finsler manifolds with three-dimensional. These structures are established on the $(M^0)^h$ and $(M^0)^v$ vector subbundles where M is an $(2n+1)$ dimensional C^∞ manifold, M^0 is a non-empty open submanifold of TM. F^* is the fundamental Finsler function and $F^{2n+1} = (M, M^0, F^*)$ is an indefinite Finsler manifold. Firstly, three-dimensional β −Kenmotsu Lorentzian Finsler manifolds are studied and some significant results are obtained. Then, three-dimensional Ricci semi-symmetric β −Kenmotsu Lorentzian Finsler manifolds are presented. Also, horizontal and vertical Ricci tensors on β −Lorentzian Finsler manifolds are calculated. As a conclusion, β −Kenmotsu Lorentzian Finsler manifolds are compared with β −Kenmotsu Lorentzian manifolds.

Keywords: *Indefinite Finsler manifolds, β −Kenmotsu manifolds, Lorentzian manifolds, Ricci tensor.*

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On C-Parallel Legendre Curves in Contact Metric Manifolds

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Abstract

In the present talk, we give the characterizations of Legendre curves in (2n+1)-dimensional non-Sasakian contact metric manifolds whose mean curvature vector fields are C-parallel or C-proper in tangent or normal bundle. Some examples of these kinds of curves are also given.

Keywords: Contact metric manifold, Legendre curve, C-parallel mean curvature vector field, C-proper mean curvature vector field.

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Compact Einstein Multiply Warped Product Manifolds

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Abstract

In the present talk, we study compact Einstein multiply warped product manifolds. We obtain the necessary and sufficient conditions for multiply warped product manifolds to be compact Einstein manifolds.

Keywords: Multiply warped product; Compact manifold; Einstein manifold.

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On the Geometric Properties of Fixed Points in Rectangular Metric Spaces

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Abstract

In the present talk, we consider some geometric properties of the set $Fix(T)$, the fixed point set of a self-mapping T on a rectangular metric space. We present new contractive conditions to obtain some fixed-disc results. All of the obtained fixed-disc results can also be considered as the fixed-circle results. We support our theoretical results with some illustrative examples.

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Keywords: Rectangular metric space, fixed circle, fixed disc, contraction.

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Gradient Yamabe Solitons on Multiply Warped Product Manifolds

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Abstract

In the present talk, we consider gradient Yamabe solitons on multiply warped product manifolds. We obtain the necessary and sufficient conditions for multiply warped product manifolds to be gradient Yamabe solitons.

Keywords: Yamabe soliton; Gradient Yamabe soliton; Multiply warped product.

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Ouasi-Einstein Manifolds with Space-Matter Tensor

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Abstract

The subject matter of this paper lies in the interesting domain of Differential Geometry and the Theory of General Relativity. To be precise the space has its motivation in Relativity, but we study its geometric properties imitating the papers on geometry regarding curvature restrictions. Such a study was joined to Einstein spaces by A. Z. Petrov. We extend it to quasi-Einstein spaces which can be considered as a generalization of Einstein spaces. This paper is supported by two examples.

Keywords: Space-Matter tensor, Einstein's field equation, Quasi-Einstein manifold.

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Reflections with respect to Line and Hyperplane by using Quaternions

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Abstract

In this study, the reflections in $\mathbb{E}^3$ and $\mathbb{E}^4$ are investigated by unit quaternions. Firstly, a linear transformation is defined to describe reflections in $\mathbb{E}^3$ with respect to the plane passing through the origin and orthogonal to the quaternion. Then some examples are given to discuss obtained results. Similarly, two linear transformations are stated which correspond to the reflection in $\mathbb{E}^4$ with respect to the hyperplane passing through the origin and a reflection with respect to the line in the direction of the quaternion. Finally, the matrix representaions of these reflections are found and the eigenvalues, eigenvectors of them are given to analyse the geometric meaning in terms of the components of the quaternion for each case.

Keywords: Quaternions; Reflections; Eigenvalues; Eigenvectors; Rigid Motions.

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A Rotation Minimizing Frame and Ruled Surface in R_1^n

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Abstract

In this paper, the pitch and the angle of pitch of a closed ruled hypersurfaces are calculated according to a Rotation minimizing frame in R_1^n.

Keywords: Rotation minimizing frame (RMF); Ruled surface; the pitch; the angle of pitch.

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Applications of Rotation Minimizing Vector Fields on Curves and Surfaces in Euclidean Space

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Abstract

In this paper, in Euclidean Space, we study the conditions of non-rotating frame and Fermi-Walker parallel according to the Fermi-Walker derivative, when a general frame is given. We also give a different perspective to the Normal Fermi-Walker derivative. In addition, we show that these Rotation minimizing vector fields are non-rotating in their linear composition according to the Fermi-Walker derivative.

Keywords: Rotation minimizing frame (RMF); Fermi-Walker Derivative; Normal Fermi-Walker Parallelism.

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Representation Varieties of 3-Manifolds and Reidemeister Torsion

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Abstract

Topological invariant Reidemeister torsion (R-torsion) was introduced by K. Reidemeister in his work [1], where he classified 3-dimensional lens spaces. W. Franz generalized this invariant and classified the higher dimensional lense spaces [2]. It has many applications in topology, differential geometry, representation spaces, knot theory, Chern-Simon theory, 3-dimensional Seiberg-Witten theory, dynamical systems, quantum fields theory and theoretical physics.

The algebraic topological instrument Symplectic chain complex was introduced by E. Witten [3], where combining this instrument and R-torsion he computed the volume of several moduli spaces Rep(Γ,G) of all conjugacy classes of homomorhisms from the fundamental group Γ of a surface Σ to the compact gauge group G=SU(2) or SO(3).

The present abstract investigates G-valued representation spaces Rep(Γ;G), where Γ is the fundamental group of 3-manifolds and G is a reductive Lie group. By using symlectic chain complex, it establishes R-torsion formulas for such representations in terms of Atiyah-Bott-Goldman symplectic form for the Lie group G. Moreover, it applies the obtained results to complete orientable hyperbolic 3-manifolds whose boundary consists of $n-$ many closed oriantable surfaces with genus at least 2 and also to Schottky representations.

Keywords: Reidemeister Torsion; Symlectic Chain Complex; Representation Varieties; 3-manifolds; Atiyah-Bott-Goldman symplectic form; Schottky representations; Thurston symplectic form; Geodesic Laminations.

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Singular Minimal Hypersurfaces

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Abstract

In this talk, we take a smooth immersion $\phi: M^n \to E^{n+1}$ of an oriented hypersurface and study the problem of finding singular minimal hypersurfaces in E^{n+1}, i.e. a hypersurface fulfilling an equation given by

$$nH = \alpha \frac{<\xi, u>}{<\phi, u>},$$

where ξ is the Gauss map of $M^n$and $u \in E^{n+1}$. Indeed, this equation means that the immersion ϕ is a critical point of the potential α-energy of ϕ in the direction u. Such an hypersurface is called singular minimal hypersurface. We obtain all singular minimal translation hypersurfaces, taking the vector u as a horizontal vector to the hyperplane $x_{n+1} = 0$.

Keywords: potential α-energy; translation hypersurfaces; singular minimal surface.

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The Equivalence Problem of Dual Parametric Curves

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Abstract

Let R be the field of real numbers and $D = \{(a, a^*) = a + \varepsilon a^*, a, a^* \in \mathbb{R}, \varepsilon^2 = 0\}$be the algebra of dual numbers. The subset $D_1 = \{(a, a^*), a \neq 0, a, a^* \in \mathbb{R}\}$of D is an abelian group with respect to the multiplication operation in the algebra D.

For an element $A = a + \varepsilon a^* \in D_1$ and a transformation $S: \mathbb{R}^2 \to \mathbb{R}^2$ where $S(A) = S_A = \begin{pmatrix} a & 0 \\ a^* & a \end{pmatrix}$, we define the sets $ID^+{}_1 = \left\{S_A = \begin{pmatrix} a & 0 \\ a^* & a \end{pmatrix}, a \neq 0, a, a^* \in \mathbb{R}\right\}$ and $ID^-{}_1 = \left\{\begin{pmatrix} a & 0 \\ a^* & a \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, a \neq 0, a, a^* \in \mathbb{R}\right\}$. Let us denote $ID_1 = ID_1^+ \cup ID_1^-$. Moreover, we denote the set $\mathscr{M}ID_1 = \mathscr{M}ID_1^+ \cup \mathscr{M}ID_1^-$ where

$\mathscr{M}ID_1^+ = \left\{F: \mathbb{R}^2 \to \mathbb{R}^2, F(B) = S_A B + C, A \in ID_1, B, C \in \mathbb{R}^2\right\}$ and

$\mathscr{M}ID_1^- = \left\{F: \mathbb{R}^2 \to \mathbb{R}^2, F(B) = (S_A W)B + C, A \in ID_1, B, C \in \mathbb{R}^2, W = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\}$.

Let $T = (a, b)$ be an open interval of $\mathbb{R}$. A $C^{(2)}$*-function* $\alpha: T \to \mathbb{R}^2$ for $\forall t \in T$ where, $\alpha(t) = (x(t), y(t))$ is called a parametrized curve (path) on the plane.

Let G be a group. Two parametric curves (paths) $\alpha(t)$ and $\beta(t)$ are called G-*equivalent* if the equality $\beta(t) = F\alpha(t)$ is satisfied for an element $F \in G$ and all $t \in T$. Then, it is denoted by $\alpha(t) \overset{G}{\sim} \beta(t)$.

This work is devoted to the solutions of problems of G-*equivalence* of parametric curves in Euclidean space $\mathbb{R}^2$for the groups $G = \mathscr{M}ID_1^+$, $\mathscr{M}ID_1$.

Keywords: Dual number, parametric curve (path), invariant.

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Conformal Slant Riemannian Maps from almost Hermitian Manifolds

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Abstract

Conformal slant Riemannian maps from almost Hermitian manifolds to Riemannian manifolds are introduced. We give an example of proper conformal slant Riemannian maps, obtain conditions for distributions to be integrable and find totally geodesicity for leaves of distributions. We also get conditions using the notion of pluriharmonicity for such maps to be horizontally homothetic maps.

Keywords: Riemannian maps; Conformal Riemannian map; Conformal slant Riemannian map.

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Cubic Surfaces and Associated Arcs

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Abstract

In 1849, Cayley and Salmon showed that a smooth cubic surface has 27 lines [3]. Later, Clebsch considered maps from the surface to the plane which are birational. This means that the map is given in each coordinate as a fraction of polynomials, and that there is an inverse map as well. These maps have the property that six disjoint lines of the surface map to points in the plane. Everywhere else, apart from a small set, the map is bijective. The surprising fact is that the six image points in the plane determine the surface up to isomorphism. In this talk, we will look at the set of nonconical six-arcs associated to some classical surfaces. Since isomorphism testing for arcs is much easier than it is for surfaces, this work contributes to the difficult question of testing when two surfaces are projectively equivalent.

The computations require the use of several computer algebra systems. Over the real numbers, Maple [8] facilitates computations in algebraic number fields. In earlier work [2], Orbiter [1] was used for the classification of surfaces. For calculations with small finite groups, GAP [5] is used.

Keywords: Geometry, Cubic Surface, Arc, Symbolic Computation, Computer Algebra

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Curvature Inequalities for Anti-invairant Riemannian Submersions from Sasakian Space form

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Abstract

In this study, we find some inequalities for anti-invariant Riemannian submersions from Sasakian space forms onto Riemannian manifolds. We obtain Chen-Ricci inequality involving the Ricci curvature and the scalar curvature for these type submersions. Equalitiy cases of these results are considered.

Keywords: Anti-invariant Riemannian submersion, Sasakian Space form, Chen-Ricci inequality

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Detecting similarities of Bézier curves for the groups LSim(E_2), LSim$^+$(E_2)

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Abstract

Let E_2 be the 2-dimensional Euclidean space, G=LSim(2) be the group of all linear similarities of E_2 and G=LSim$^+$(2) be the group of all orientation-preserving linear similarities of E_2.

In [1], using *local* differential invariants and Frenet frames of two curves, uniqueness and existence theorems for a curve determined up to a direct similarity of E_n.

For the group Sim$^+$(n), this theorem shows that a necessary and sufficient conditions for two curves in E_n to be equivalent is that they have same shape curvatures and the other specially conditions.

In [2], The complete systems of global G-invariants of a path and a curve in E_2 are obtained. For the groups G, existence and uniqueness theorems for a curve and a path are given in terms of *global* G-invariants of a path and a curve.

In [3], LSim(2)-equivalence of two Bézier curves without using differential invariants of Bézier curves in terms of control invariants of Bézier curves is proved.

In this work, starting from the ideas in [2], [3] and [4] , we address how to compute explicitly an linear similarity transformation which carrying a Bézier curve into another Bézier curve in terms of control invariants of a Bézier curve for the groups LSim(2) and LSim$^+$(2) without using differential invariants of Bézier curves.

Keywords: Bézier curve, linear similarity, invariant.

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Smarandache Curves According to q-Frame in Minkowski Space

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Abstract

In this study, we investigate special Smarandache curves according to q-frame in Minkowski space and we give some differential geometric properties of Smarandache curves.

Keywords: Frenet frame; Smarandache curves; q-frame; Natural curvatures.

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Slant Curve in Lorentzian Bianchi -Cartan-Vranceanu Geometry

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Abstract

In this study, we investigate slant curves for the Lorentzian Bianchi-Cartan-Vranceanu metric. We show a simpler form of directional derivative on the Lorentzian Bianchi - Cartan - Vranceanu metric. We define definitions of non-lightlike and non-geodesic slant curves in Lorentzian BCV manifolds. Moreover we write and prove some theorems with respected to slant curves in Lorentzian BCV manifolds. Moreover, we investigate the spherical indicatricies of slant helices in these spaces. we investigate pherical images the tangent indicatrix, principal normal indicatrix and binormal indicatrix of slant helices in Lorentzian BCV manifolds. We obtain some results about them.

Keywords: Bianchi-Cartan-Vranceanu metric; Lorentzian metric; slant curve; spherical indicatricies.

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Ruled Surfaces with Constant Slope Ruling with Quaternionic Representations

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Abstract

In this study, we investigate ruled surfaces with a constant slope ruling with respect to the osculating, rectifying and normal surfaces and examined many features of these surfaces. At the same time, we give some geometric properties of these surfaces such as striction curves, Gaussian and mean curvatures. Finally, we give surfaces definitions with quaternionic representations. Also, we obtain some of the results and present some necessary conditions for surfaces to be flat or minimal surfaces. Then, we illustrate some examples of the surfaces.

Keywords: Ruled Surfaces; Surface with Constant Slope; Quaternions.

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Bi-Slant Submersions in Paracomplex Geometry

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Abstract

We introduce the notion of bi-slant submersions from para-Kaehler manifolds onto pseudo-Riemannian manifolds. Naturally, they englobe semi-slant and hemi-slant submersions. We study their fist properties and a whole gallery of examples.

Keywords: Para-Kaehler manifold; pseudo-Riemannian submersion; bi-slant submersion.

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Spacelike Curves and B_2 −Slant Helices in R_2^4

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Abstract

Let α be a spacelike curve in R_2^4, parametrized by arclength function of s. The following cases occur for the spacelike curve α. Let the vector N is spacelike, B_1 and B_2 be timelike. In this case there exists only one Frenet frame {T,N, B_1, B_2 } for which $\alpha(s)$ is a spacelike curve with Frenet equations

$$\nabla_T T = k_1 N$$
$$\nabla_T N = -k_1 T + k_2 B_1$$
$$\nabla_T B_1 = k_2 N + k_3 B_2$$
$$\nabla_T B_2 = -k_3 B_1$$

In this paper by establishing the Frenet frame {T,N,B_1,B_2} for a spacelike curve we give some characterizations for the spacelike inclined curves and B_2-slant helices in R_2^4.

Keywords: B_2-slant helix, inclined curve, spacelike curve.

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Modified Spinorial Levi−Civita Connection on the Spin Hypersurfaces of Manifolds

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Abstract

In this paper, we give an estimates for the eigenvalues of the hypersurfaces Dirac operators by using a modified spinorial Levi−Civita connection. Then, by considering limiting case we show that the hypersurface is an Einstein.

Keywords: Spin and Spin^c geometry; Dirac operator; Estimation of eigenvalues.

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Rotational Weingarten Surfaces in 3-Dimensional Space Forms

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Abstract

We study rotational Weingarten surfaces in the 3-dimensional space forms with the principal curvatures κ and λ satisfying a certain functional relation $\kappa = F(\lambda)$ for a given continuous function F. We determine profile curves of such surfaces parameterized in terms of the principal curvature λ. Then we consider some special cases by taking $F(\lambda) = a\lambda + b$ and $F(\lambda) = a\lambda^m$ for particular values of the constants a, b and m.

Keywords: Rotational surfaces; Weingarten surface; Mean curvature; Gaussian curvature.

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On a Class of Hypersurfaces in Euclidean Spaces with Zero Gauss-Kronecker Curvature

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Abstract

In this work, we study a class of hypersurfaces in a Euclidean space of the dimension $n+1$ for a given $n \in \mathbb{N}$. The principle curvatures of these hypersurfaces are $k_1 = 0$ and $k_2 = k_3 = \cdots = k_{r+1} = c$ for a non-zero constant c and $r \in \{1,2,\ldots,n-2\}$. Consequently, the Gauss-Kronecker curvature of these hypersurfaces vanishes identically. We obtain some of other important geometrical properties of these class of hypersurfaces.

Keywords: Hypersurfaces, Euclidean spaces, Gauss-Kronecker curvature, constant principle curvatures

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On Special Curves of General Hyperboloid in E^3

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Abstract

In this study, it is investigated the curve whose its position vector fields makes a constant angle with the constant vector fields on the general hyperboloid. In order to calculate these curves, it is utilized the hyperbolically motion and hyperbolically inner product which are defined by Simsek and Özdemir in [4]. Also, some examples of them are plotted by using the Mathematica program.

Keywords: General hyperboloid; Special curves; Euclidean space; Darboux frame.

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Spacelike and Timelike Constraint Manifolds for A Closed Chain on Lorentz Plane

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Abstract

The movement at each joint to the position of the last link is associated with the structure equation of an open chain. The structure equation for single loop closed chain is obtained by using the structure equation of the open chain. The constraint manifold of a closed chain is found if the structure equation for this chain is taken into consideration. In this study, making use of the structure equations of a planar open chain in Lorentz space, we present the structure equations for a 4R closed chain on Lorentz plane. Then, using these structure equations, the constraint manifolds for closed chain on Lorentz plane are attained.

Keywords: Lorentz Planar Displacement; Close Chain; Contraint Manifold; Structure Equations.

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A Characterization of Weak Biharmonic Rotational Surfaces in E^4

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Abstract

Minimal surfaces in Euclidean spaces are important subject in differential geometry. Biharmonic surfaces are the generalization of minimal surfaces. Meanwhile, weak biharmonic surfaces are another consideration of surfaces. In the present study, we consider weak biharmonic rotational surfaces in Euclidean 4-space E^4. In this consideration we found some results of general rotational surfaces, spherical product surfaces and meridian surfaces in E^4 of weak biharmonic type. We also give some examples.

Keywords: Biharmonic; Mean curvature; Rotational surfaces.

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Some Tensor Conditions of Globally Framed Almost f – Cosymplectic Manifolds

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Abstract

In this study we get some classifications of globally framed almost f-cosymplectic manifolds under some tensor conditions. Also we give some results on η-parallelity, cyclic parallelity, Codazzi condition. Finally, we give an explicit example of globally framed almost f-cosymplectic manifolds.

Keywords: Framed manifold, Kenmotsu manifold, Cosymplectic manifold.
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A General Fixed Point Theorem on A-Metric Spaces

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Abstract

In this work, we will briefly talk about the expansion of metric spaces starting from the definition of the ordinary metric space to the present day. Afterwards, we will present A-metric spaces and some basic properties of its. Finally, we will give a general fixed point theorem in A-metric space. This fixed point theorem contains many of the well-known fixed point theorems from ordinary metric spaces to S-metric spaces as the applications.

Keywords: Generalized metric space; S-metric space; A-metric space; Fixed point theory.

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On Obtaining Complete S-Metric Space

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Abstract

The aim of this work is to show that for a mapping F defined on a complete $S-$metric space (X,S), if F is not a contraction mapping but a power of F ($F^n, for\ n \in IN^+$) is a contraction mapping, then there exist a related (X,S) another complete $S-$metric space such that F is a contraction mapping on this space.

Keywords: Metric spaces; Generalized metric space; S-metric space; A-metric space; Fixed point theory, Metric geometry.

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On The Geometry of Submanifolds of a (k, μ)-Contact Manifold

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Abstract

The object of this paper is to study submanifolds of (k,μ)-contact manifolds. We find the necessary and sufficient conditions for a submanifold of (k,μ)-contact manifolds to be invariant and anti-invariant. Also, we research the necessary and sufficient conditions for a submanifold of a (k,μ)-contact to be pseudo-parallel and semi-parallel submanifold and get interesting results.

Keywords: (k,μ)-contact manifold, Invariant Submanifold, Anti-Invariant Submanifold, Pseudo–Parallel Submanifold and Semiparallel Submanifold.

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The Geometry of Complex Metallic Conjugate Connections

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Abstract

In this study, we give some properties of the conjugate connection on a complex metallic structure. We express the complex metallic conjugate connections in terms of structural and virtual tensors from the almost complex structure. In addition, the existence of duality between the complex metallic and almost complex conjugate connection is investigated.

Keywords: Complex metallic structure; (Conjugate) Linear connection; Almost complex manifold; Structural and virtual tensor field.

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Timelike V-Bertrand Curve Mates in Minkowski 3-Space

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Abstract

Camcı defined new type Bertrand curve in E^3. The new type Bertrand curve is said V-Bertrand curve. In this paper, we study V-Bertrand curve in Minkowski 3-space. Also the characterization of the timelike V-Bertrand curve, the distance between the opposing points of the Timelike V-Bertrand curve pairs and the angle between the tangent vectors was examined. Timelike f-Bertrand curve and timelike Bertrand surface definitions were given and supported by examples. In the last section, timelike Bertrand and principal-donor curves were constructed. In addition, the Salkowski method was restructured and the necessary characterization was given to obtain the new timelike Bertrand curves.

Keywords: Bertrand Curve; Timelike V-Bertrand Curve; Minkowski 3-Space.

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Timelike V-Mannheim Curve Mates in Minkowski 3-Space

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Abstract

Camcı defined a new type Mannheim curve, called V-Mannheim curve in Euclidean 3-space. In this work , we study on V-Mannheim curve in Minkowski 3- Space. Especially, We focus on timelike case of these curves . Additionally , we give a characterization of timelike V-Mannheim curve in Minkowski 3-Space.

Keywords: V-Mannheim curve ; Minkowski 3- Space ; Mannheim curve

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An Example of Curvatures of a Sliced Contact Metric Manifold

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Abstract

We defined sliced almost contact metric manifolds as a wider class of almost contact metric manifolds in my PhD thesis. In this work we calculated the ϕ_π-sectional curvature and the Riemannian curvature tensor of the sliced almost contact metric manifolds. Hence we think that all these studies will accelerate the studies on the contact manifolds and their submanifolds.

Keywords: Contact geometry, Sectional curvature, Riemannian curvature, Sliced almost contact metric manifolds, Sliced contact metric manifolds.

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A Study on Timelike Directional Bonnet Canal Surfaces

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Abstract

A canal surface is defined as an envelope of a one-parameter family of spheres. In this paper, we give a parametric representation of a timelike directional canal surface in the Minkowski space using q-frame. Later, the case of orthogonality of parameter curves are investigated and some geometric properties of timelike directional Bonnet canal surfaces are given. Finally, some examples of timelike directional Bonnet canal surfaces with q-frame are constructed and plotted.

Keywords: q-frame; Canal Surface; Bonnet Surface.

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Spherical Indicatrices of Directional Space Curve

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Abstract

In this paper, a directional space curve by using adapted frame called q-frame is introduced. We first investigate the spherical indicatrices of this space curve. Then we work on the conditions that a space curve to be helix and slant helix by using the geodesic curvature of the spherical image of the directional tangent and normal indicatrices, respectively. Finally, some applications of the results are given.

Keywords: q-frame; Spherical Indicatrix; Slant Helix .

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Some Notes on Poly-Norden Manifold

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Abstract

The paper deals with a almost poly-Norden manifolds. We investigated integrability properties of the almost poly-Norden manifold with a special opetaror. Then, we have define special metric connection and present some examples on this manifold.

Keywords: Poly-Norden manifolds; metric connections.

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Smarandache Curves of Spacelike Salkowski Curve with a Spacelike Principal Normal According to Frenet Frame

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Abstract

In this study, we investigate TN, TB, NB and $TNB-$Smarandache curves of spacelike Salkowski curve with a spacelike principal normal according to Frenet frame. firstly, we definite TN, TB, NB and $TNB-$Smarandache curves depending upon the Salkowski curve. later, the curvature and the torsion Frenet vectors of Smarandache curves are calculated. Finally, we draw graphic of the obtained Smarandache curves and some related results are given.

Keywords: Minkowski space; Spacelike Salkowski curve; Spacelike Smarandache curve.

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Instantaneous Kinematics of a Planar Two-Link Open Chain in the Complex Plane

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Abstract

This paper base on the complex-number method for the aim of studying the instantaneous kinematics of the terminal link of a planar two-link open chain using the canonical coordinate system of planar two-parameter motion. We review the higher order instantaneous invariants of this motion of the terminal link with reference to the first and second order instantaneous invariants. We apply these instantaneous invariants to curvature theory. Finally, we give some examples by choice of different parameters and exhibit the comparisons.

Keywords: Instantaneous invariants; Curvature Theory; Planar Two-Parameter Motion.

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Some Fixed Point Theorems in G-Metric Spaces with Order n

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Abstract

In this work, we will talk to present briefly G-metric spaces with order n. Afterwards, we will give fundamental properties, several examples, and topological properties on the g-metric space including the convergence of sequences and the continuity of mappings on the G-metric space with order n. Finally, we will give some fixed point theorems on G-metric spaces with order n. These fixed point theorems are extension of well-known fixed point theorems in G-metric spaces.

Keywords: Generalized G-metric space; G-metric space; Fixed point theory.

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Loxodromes on Space-like Rotations Surfaces in E_1^4

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Abstract

In this talk, the differential equations of loxodromes on space-like rotational surfaces in Minkowski 4-space E_1^4 are investigated. So far, since there is no study about loxodromes on space-like rotational surfaces in E_1^4, it is expected that results of this study will give us several contributions about navigation.

Keywords: Loxodromes, Space-like Rotations Surfaces, Minkowski 4-Space.
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Loxodromes on Time-like Rotations Surfaces in E_1^4

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Abstract

In this talk, the differential equations of time-like and space-like loxodromes on time-like rotational surfaces in Minkowski 4-space E_1^4 will be investigated and the lengths of the loxodromes will be calculated. Since the equations of loxodromes on the time-like rotational surfaces in Minkowski-4 space have not been calculated so far, the results will be contributed to a better understanding of the structures in the Minkowski-4 space and to the navigation studies.

Keywords: Loxodromes, Time-like Rotations Surfaces, Minkowski 4-Space.

This paper was supported by the Scientific Research Project Coordination Unit of Yozgat Bozok University under Project 6601-FBE/19-279.

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On Some Geometric Properties of Contact Pseudo-Slant Submanifolds of a Sasakian Manifold

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Abstract

The object of this paper, we study geometry of the contact pseudo-slant submanifolds of a Sasakian manifold. We derive the integrability conditions of distributions in the definition of a pseudo-slant submanifold. The notions contact pseudo-slant product and contact parallel are defined and the necessary and sufficient conditions for a submanifold to be contact pseudo-slant product and contact parallel are given.

Keywords: Sasakian manifold; Sasakian space form; contact slant submanifold; contact pseudo-slant submanifold.

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On C-Bochner Curvature Tensor in $(LCS)_n$-Manifolds

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Abstract

In [5] S. Bochner defined the Bochner curvature tensor on a Kahler manifold. This tensor is constructed formally by modifying Weyl's conformal curvature tensor. In [2], Endo defined E-Bochner curvature tensor as an extended C-Bochner curvature tensor. Recently, many geometers have been concerned with Bochner's tensor and in particular they studied Kahler manifolds with vanishing Bochner curvature tensor. The aim of this paper is to study the C-Bochner curvature tensor in $(LCS)_n$-manifolds.

Keywords: Bochner curvature tensor; $(LCS)_n$-manifold; Projective.

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Involute Curves in 4-dimensional Galilean space G_4

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Abstract

Galilean geometry is one of the nine geometries of projective space which was discussed by Cayley-Klein at the beginning of 20th century. After that, the curvature-related studies were maintained and the curve properties in Galilean space were studied in [1]. The involute of a given curve is a well-known concept in Euclidean space, whereas the idea of an involute string is due to C. Huygens, who is well-known for his work in optics and has discovered involutes while trying to build a more precise clock in 1968 [2].

In classical differential geometry, an evolute of a curve is defined as the locus of the centers of curvatures of the curve, which is the envelope of the normal of the given curve. While an Involute of a given curve is a curve to which all tangents of a given curve are normal [2].

In this paper, we define the (0,2)-involute of a given curve in 4-dimensional Galilean space, and for the curve with a generalized involute, the necessary and sufficient condition is obtained.

Keywords: Galilean space, Involute curve, Frenet formula.

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The Generalized B-Curvature Tensor on Normal Paracontact Metric Manifold

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Abstract

In 2014, Shaikh and Kundu [1] introduced and studied a type of tensor field, called generalized B curvature tensor on a Riemannian manifold. This includes the structures of Quasiconformal, Weyl conformal, Conharmonic and Concircular curvature tensors. The aim of this paper is to study the generalized B-curvature tensor of a normal paracontact metric manifold. Moreover we consider the conditions generalized B-flat, generalized B-semi symmetric, B.S=0 and B.Z=0.

Keywords: Generalized B-curvature; Normal Paracontact; Concircular.

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Notes on Second-Order Tangent Bundles

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Abstract

In this paper, we consider a second-order tangent bundle equipped with Sasaki metric over a Riemannian manifold. All forms of curvature tensor fields are computed. We obtained the relation between the scalar curvature of the base manifold and the scalar curvature of the second-order tangent bundle. Finally, we presented geometric results concerning kinds of curvature tensor fields.

Keywords: Second-order tangent bundle; curvature tensor field; lifts.
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Space-like Loxodromes on Helicoidal Surfaces in E_1^4

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Abstract

In this talk, we first study a class of helicoidal surfaces in Minkowski space-time E_1^4. After that, we obtain the equations of space-like loxodromes on the non-degenerate helicoidal surfaces in E_1^4.

Keywords: Loxodrome; Helicoidal Surface; Minkowski Space-Time.

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Some Results on Rectifying Direction Curves in E^3

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Abstract

In this paper, we introduce a new type of special curves in 3-dimensional Euclidean space. We give the characterizations for these curves and we show that rectifying-direction curve and rectifying donor curve constitute a Bertrand pair. We also explain the achieved results with examples.

Keywords: Associated curves; Rectifying-direction curves; Rectifying-donor curves.

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On Directional Curves in 3-Dimensional Minkowski Space

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Abstract

In this paper, we consider special curves generated via directional curves in 3-dimensional Minkowski space. We obtain some relations between the directional curves and the generated curves by helped Frenet apparatus. Also, we calculate curvatures of these curves and give some conclusions.

Keywords: Associated curves; Directional curves; Donor curves.

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Some Lift Problems in Semi-tensor Bundle of Type (p,q)

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Abstract

We define a semi-tensor bundle tM of type (p,q) with respect to projection of the tangent bundle TM over a manifold M. The main purpose of the present paper is to study some vertical lifts of tensor fields and complete lift problems of vector fields to semi-tensor (pull-back) bundle tM of type (p,q).

Keywords: Complete Lift, Pull-back Bundle**,** Tangent Bundle**,** Semi-tensor Bundle.

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Some Results on Metric Contact Pairs

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Abstract

Blair, Ludden and Yano [2] introduced the notion of bicontact in the context of Hermitian geometry. In 2000's Bande and Hadjar [3-8] study on this notion under the name of contact pairs. They gave results on the Riemannian geometry of metric contact pairs. These type of structures have important properties and their geometry is some different from classical contact structures. In this study we present general properties of metric contact pairs and we obtain some results under certain curvature conditions.

Keywords: Contact metric pair; bicontact; curvature properties.

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Historical and Philosophical Foundations of non-Euclidean Geometry

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Abstract

Flat geometry, or the Euclidean geometry as we call it, despite its certain values to understand the universe, still has some puzzling features. One of these features is the fifth postulate of Euclid. By considering the impossibility of proving that postulate, mathematicians revealed the notion of a non-Euclidean geometry. This form of geometry is thought to be able to reveal certain enigmas of the universe. And as well as having a history of mathematical developments behind it, non-Euclidian geometry also has very deep philosophical foundations. In this study, we reflect upon these historical and philosophical foundations of non-Euclidean geometry, through the works of Gauss and Riemann in particular.

Keywords: non-Euclidean geometry; philosophical foundations of geometry; Riemann; geometry history

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A Study on Directional Generalized Tubes

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Abstract

In this paper, we consider generalized tubes, which we refer to in the paper as hereafter GTs, according to q frame in Euclidean space E^3. First, we give a parametric representation of directional generalized tubes (DGTs). Since GT class is divided by two important subclasses, we investigate geometric properties of these two classes with respect to the q-frame.

Keywords: Frenet frame; Generalized tubes; Adapted frame.

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On k-type Slant Helices due to Bishop Frame in Euclidean 4-space E^4

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Abstract

In this work, we study k-type (k=0,1,2,3) slant helices with non-zero Bishop curvature functions due to Bishop frame in E^4. General helix is a 0-type slant helix within the notation of this study. We characterize all of slant helices in terms of Bishop curvatures in E^4

Keywords: Bishop Frame; General Helix; Slant Helix; Euclidean 4-Space.

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New Version of Integral Representation Formula in Bianchi Type-I Spacetime

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Abstract

In this work, we obtain a method to derive a Integral-type representation formula for simply connected immersed surfaces in Bianchi type-I spacetime. We use the left invariant metric and obtain some results of Levi-Civita connection. Furthermore, we show that any harmonic map of a simply connected coordinate region D into Bianchi type-I spacetime can be represented a form.

Keywords: Bianchi type-I (B-I) cosmological model; Integral representation formula.

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Galilean Transformation for Bertrand Curves of Biharmonic Curves in Heisenberg Group

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Abstract

In this paper, we characterize Galilean transformation of Bertrand curves of biharmonic curves in the Heisenberg group $Heis^3$. Finally, we find explicit parametric equations of Galilean transformation of Bertrand curves of biharmonic curves in the Heisenberg group $Heis^3$.

Keywords: Galilean relativity; biharmonic curves; bienergy; Heisenberg group, symmetries.

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An Approach for on Π_1-Surfaces of Biharmonic Constant Π_2-Slope Curves According to Type-2 Bishop Frame in The Sol Space

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Abstract

In this paper, we study Π_1- surfaces of biharmonic constant Π_2- slope curves according to type-2 Bishop in the SOL^3. We characterize asymptotic curves on Π_1- surfaces of biharmonic constant Π_2- slope curves in terms of their Bishop curvatures. Finally, we find out their explicit parametric equations in the SOL^3.

Keywords: Sol Space; Curvatures; Asymptotic curve; Bienergy

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Bonnet Surfaces of Integrable Geometric Flows with Schrödinger Flow

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Abstract

In this work, we obtain a new approach for computing the differential geometry properties of surfaces by using Bäcklund transformations of integrable geometric curve flows. Moreover, we have conditions of Bonnet surfaces of Schrödinger flow. Finally, we give some new solutions by using the extended Riccati mapping method for Schrödinger flow. Finally, we obtain figures of this solutions.

Keywords: Bäcklund transformations; Schrödinger flow; Extended Riccati mapping method.

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Inextensible Flows of Principal-Direction Curves in Euclidean 3-Space

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Abstract

In this paper, we construct a new method for principal-directioncurves of inextensible flows of curves in $\mathbb{E}^3$. Using the Frenet frame of the given curve, we present partial differential equations. We give new characterizations for curvatures of a curve in $\mathbb{E}^3$.

Keywords: Inextensible flows, principal-direction, associated curve.

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New Approach for Inextensible Flows of Π_1 Bishop Spherical Images According to Type-2 Bishop Frame

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Abstract

In this paper, we study Π_1 Bishop spherical images in Euclidean space E^3. Using the type-2 Bishop frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in Euclidean space E^3.

Keywords: Type-2 Bishop frame; Space; Curvatures; Flows.

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On Focal Curve of Spacelike Curve According to Modified Frame

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Abstract

In this work, we obtain a new characterization of focal curves of spacelike curves with respect to modified orthogonal frame in Minkowski 3-space. Finally, the correlation between the focal curvature and the radius of the sphere S_1^2 is given.

Keywords: Modified orthogonal frame; spacelike curve; Minkowski space; focal curve

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New Focal Curves of Timelike Curves According to Ribbon Frame in Minkowski Space

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Abstract

In this paper, we characterize focal curves of timelike curves according to Ribbon frame in the Minkowski 3-space. We construct its focal curves in terms of their focal curvatures.

Keywords: Ribbon frame; Minkowski 3-space; Focal curve

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On Bihyperbolic Numbers and Their Geometric Properties

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Abstract

In this study, we briefly mention the hyperbolic numbers and the bihyperbolic numbers which are a kind of the commutative quaternion. We give two different idempotent representations of byhiperbolic numbers. Afterwards, we set off on a quest for the relationship between bihyperbolic numbers with $\mathbb{R}_2^4$ semi-Euclidean space. For this purpose, we define three norms of a bihyperbolic number. Thus, we obtain new results for these norms and define space cone, null cone and time cone at a point in $\mathbb{R}_2^4$ by these new defined norms.

Keywords: Bihyperbolic Numbers; Norm of Bihyperbolic numbers; Topologies of Bihyperbolic Space.

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Some Suborbital Graphs Drawn on The Poincare Disc

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Abstract

The idea of suborbital graphs has been used mainly by finite group researchers in all references. In [4], Jones et al. investigated suborbital graphs which are extension of Farey graphs with imprimitive action of the modular group on the set of extended rationals. Similarly, in this paper we investigate suborbital graphs of a special congruence subgroup of the modular group. While most of the studies are shown on the upper half plane, in this study we show the graphs on Poincare disk.

Keywords: Circuit; Imprimitive Action; Suborbital Graphs.

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On The Variational Arcs due to ED-Frame Field in Euclidean 4-Space

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Abstract

In this study, we define a variational field for constructing a family of Frenet curves of the length l lying on an oriented hypersurface and calculate the length of the variational arcs due to ED-frame field in Euclidean 4-space. And then, we derive the intrinsic equations for the variational arcs and also obtain boundary conditions for this type curves due to ED-frame field in Euclidean 4-space.

Keywords: Euclidean 4-space; ED-frame field; Variational arc.

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On Darboux Helices in the Complex Space C^3

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Abstract

In this study, we define the notion of Darboux helix for isotropic curves in 3-dimensional complex space C^3. We show that every isotropic curve with constant pseudo curvature is A Darboux helix. Also we find the axis of isotropic Darboux helix.[3.]

Keywords: Isotropic curve; Pseudo curvature; Isotropic Darboux helix.

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Codazzi Couplings of Riemannian Manifolds with a Structure of Electromagnetic Type

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Abstract

Let M be a Riemannian manifold equipped with a structure of electromagnetic field J, a compatible Riemannian metric g, a torsion free connection ∇. In this paper, we study Codazzi couplings on the Riemannian manifolds, such as Codazzi coupling of ∇ with J, Codazzi coupling of ∇^* with J, Codazzi coupling of ∇^+ with J and Codazzi couplings of ∇^* and ∇^+ with G, where ∇^* is g-conjugate connection, ∇^+ is G-conjugate connection.

Keywords: Codazzi coupling, Electromagnetic field , Conjugate connection.

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The Study of Pseudo Symmetry of a Normal Complex Contact Space Form

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Abstract

A locally symmetric normal complex contact metric manifold is locally isometric to the complex projective space with the Fibini-Study metric [4]. In this talk we show that is not possible for a normal complex contact space form to be properly pseudo symmetric, unlike the real case [1].

Keywords: Complex contact manifold; complex contact space form; pseudo-symmetry.

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On Suborbital Graphs with Hyperbolic Geodesics and Entries of Matrices from Some Sequences

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Abstract

In this study, we investigate the values of the special vertices of the suborbital graph $\mathbf{F}_{u,N}$ and relation between even index terms of famous sequences such as Fibonacci and Lucas. From these relations, we get some new results to have the terms of these sequences. Also we get some connections between the values of these special vertices and matrices consisting of even index terms of these sequences.

Suborbital graphs are formed by imprimitive action, which is the action of a congruence subgroup of the Modular group Γ on the extended rational set $\widehat{\mathbb{Q}} := \mathbb{Q} \cup \{\infty\}$. These graphs are Γ-invariant directed graphs and their vertices are from the set $\widehat{\mathbb{Q}}$ and their edges are from the set $\widehat{\mathbb{Q}}^2$ as hyperbolic geodesics in the one type of model of hyperbolic geometry, which is the upper half plane $\mathbb{H} := \{z \in \mathbb{C} : Im(z) > 0\}$.

We also give some results by using some properties of the suborbital graph $\mathbf{F}_{u,N}$ from [1] with these special sequences.

Keywords: Suborbital Graphs; Modular group; Periodic Continued Fractions; Fibonacci Sequence; Lucas Sequence.

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The Farthest Vertices on the Suborbital Graphs via Hyperbolic Geometry

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Abstract

In this study, we investigate the farthest vertex where a vertex can be connected on the path of minimal length on the suborbital graph $\mathbf{F}_{u,n}$. The values of these special vertices are based on periodic continued fractions and derived by an element of the congruence subgroup of the Modular group Γ.

The elements of Γ sends the hyperbolic lines to hyperbolic lines. So, we have represented the edges of graphs as hyperbolic geodesics in the upper half plane

$$\mathbb{H} \coloneqq \{z \in \mathbb{C} : Im(z) > 0\},$$

which is the one model of hyperbolic geometry. Hyperbolic lines are as euclidean semi-circles or half-lines perpendicular to $\mathbb{R}$ as in [7].

We also give some results by using some properties of the suborbital graph $\mathbf{F}_{u,n}$ from [1] with these special continued fractions.

Keywords: Suborbital Graphs; Modular group; Periodic Continued Fractions.

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On Construction of Q-Focal Curves in Euclidean 3-Space

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Abstract

In this paper, we study Q−focal curves in the Euclidean 3-space E^3. We characterize Q−focal curves in terms of their focal curvatures.

Keywords: Focal curve; Q-frame; Euclidean 3-space.

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On Design Developable Surfaces according to Quasi Frame

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Abstract

A developable surface is a ruled surface having Gaussian curvature K = 0 everywhere. Developable surfaces therefore include the cone, cylinder, elliptic cone, hyperbolic cylinder, and plane. By utilizing the Quasi frame, this paper proposes a new method to construct a developable surface possessing a given curve as the line of curvature of it. We analyze the necessary and sufficient conditions when the resulting developable surface is a cylinder, cone or tangent surface.

Keywords: Quasi frame, Developable surface cylinder surfaces, cone surfaces.

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Dual Generalized Quaternions and Spatial Kinematics

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Abstract

In this work, finite spatial displacements and spatial screw motions were given by using generalized dual quaternions and Hamiltonian operators. Relative motion in 3-dimensional generalized dual sphere was expressed by Hamiltonian operators of generalized dual quaternion. We gave the relation between Hamiltonian operators and transfromation matrices. Morever, spatial displacement were given by screw motion in 3-dimensional generalized space $IR^3_{\alpha\beta}$..

Keywords: Generalized Dual Quaternion, Screw Motion, Hamiltonian operators, Lie Algebra

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Cayley Formula, Euler Parameters and Rotations in Generalized Quternions

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Abstract

In this work, we obtained Cayley formulae of orthogonal matrice and Euler parameters in 3-dimensional generalized space $IR^3_{\alpha\beta}$. Afterwards, by using Euler parameters of a rotation in a generalized quaternion, the equation of generalized quaternion of rotation movement was obtained in 3-dimensional generalized space $IR^3_{\alpha\beta}$.

Keywords: Generalized Dual Quaternion, Screw Motion, Hamiltonian operators, Lie Algebra

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A Note on Hypersurfaces of Almost poly-Norden Riemannian Manifolds

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Abstract

In the present paper, we introduce hypersurfaces of almost-poly Norden Riemannian manifolds. We investigate conditions for a hypersurface of an almost poly-Norden Riemannian manifold to be invariant and totally geodesic, respectively, in terms of the components of the structure induced by almost poly-Norden structure on the ambient manifold. We also obtain some results for totally umbilical hypersurfaces and give examples.

Keywords: Bronze Mean; Poly-Norden Manifold; Invariant Hypersurface.

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Biharmonic Curves in 3-dimensional *f*-Kenmotsu manifolds

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Abstract

In this study, we investigate necessary and sufficient conditions for a slant curve in 3-dimensional *f*-Kenmotsu manifold to be biharmonic. We also give some characterizations for the biminimality of such curves.

Keywords: Biharmonic curve, biminimal curve, *f*-Kenmotsu manifold.

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Some Results on Bi-f-Harmonic Curves in (α,β)-Trans Sasakian Generalized Sasakian Space Forms

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Abstract

In this paper, we investigate bi-f-harmonicity of Legendre curves in (α,β)-trans Sasakian generalized Sasakian space forms.

Keywords: Bi-f-harmonic curve, Legendre curve, (α,β)-trans Sasakian generalized Sasakian space forms.

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On a Type of Lightlike Submanifold of a Golden Semi-Riemannian Manifold

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Abstract

In this article, we examine the term of screen pseudo-slant lightlike submanifolds of a golden semi-Riemannian manifold. Also, we obtain an example. We give some characterizations about the geometry of such submanifolds.

Keywords: Semi-Riemannian manifold, Golden ratio, Lightlike submanifold.

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Ruled Surfaces whose Base Curves are Non-Null Curves with Zero Weighted Curvature in E_1^3 with Density e^{ax+by}

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Abstract

In this study, the weighted curvatures of spacelike and timelike planar curves in E_1^3 with density e^{ax+by} are given and the curves with zero weighted curvatures in Lorentz-Minkowski space with density e^{ax+by} according to the cases of constants a and b are obtained. Also, the ruled surfaces whose base curves are spacelike planar curves with zero weighted curvature in Lorentz-Minkowski space with density e^{ax} and the ruling curves are its Smarandache curves are investigated and some characterizations have been given for these surfaces.

Keywords: Weighted curvature, Lorentz-Minkowski space, Spacelike and timelike curves, Ruled surface.

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Rotational Surfaces Generated by Non-Null Curves with Zero Weighted Curvature in E_1^3 with Density $e^{ax^2+by^2}$

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Abstract

In this paper, rotational surfaces which are generated by spacelike and timelike curves with zero weighted curvatures in Lorentz-Minkowski space E_1^3 with density $e^{ax^2+by^2}$ are studied according to some cases of not all zero constants a and b.

Keywords: Weighted curvature, Lorentz-Minkowski space, Spacelike and timelike curves, Rotational surface.

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On the Curves N - $T^{\times}N^{\times}$ in E^3

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Abstract

Evolute and involute curves, Manheim curves are given as the famous examples fort he offset curves. Also Bertrand curves are another example, to produce new curves based on the other curves with common principal normal vector fields. Before we examined $ND^{\times}$curve with common principal normal vector of first curve and Darboux vector of the second curve. In this paper we have defined, and examined the new kind curves, with the principal normal vector field of the first curve and the vector field, which lies on the osculator plane of the second curve are linearly dependent. As a result we have called these new curves as N - $T^{\times}N^{\times}$ curves. Also similiar to the other offset curves, Under the spesific condition, we give Frenet apparatus of the second curve based on the Frenet apparatus of the first curve.

Keywords: Offset curves; Mannheim curves; Bertrand pairs

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Null Cartan Curves of Constant Breadth

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Abstract

The curves of constant breadth are special curves, which have a wide range of application. In studies conducted so far, some integral characterizations of these curves have been obtained [2, 3]. In this study, firstly differential equations characterizing Null Cartan curves of constant breadth are obtained. These equations are 3rd order, linear differential equations with variable coefficients. These type equations are generally impossible to solve analytically and so, for approximate solution we presented a new numerical method based on hermite polynomials by using initial conditions. We call this technique the modified Hermite matrix-collocation method [1]. In addition, with the help of these solutions, some geometric properties of this curve type are examined.

Keywords: Null Cartan curves; Hermite matrix method; Constant breadth.

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On Quaternionic Space Curves of Constant Breadth

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Abstract

The curves of constant breadth are special curves used in engineering, architecture and technology. In the literature, these curves are considered in different spaces according to different roofs and some integral characterizations of these curves have been obtained [1,2,3]. However, in order to examine the geometric properties of curves of constant breadth, more than characterization is required. In this study, firstly differential equations characterizing quaternionic space curves of constant breadth are obtained. Then, approximate solutions of the obtained differential equations are calculated. The geometric properties of this curve type are examined with the help of these solutions.

Keywords: Curve of constant breadth; Quaternionic space curve.

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On Some Characterizations of the Harmonic and Harmonic 1-Type Curves in Euclidean 3-Space

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Abstract

In our study, we calculated the characterizations of space curves according to N-Bishop frame in Euclidean 3-space. Moreover, we examined some differential equation characterizations of the harmonic and harmonic 1-type curves and gave some results regarding condition of the helix.

Keywords: Harmonic curve; Harmonic 1-type curve; N-Bishop frame.

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On the Curvatures of Tangent Bundle of a Hypersurface in E^{n+1}

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Abstract

Let M be an immersed orientable hypersurface $f: M \subset \mathbb{R}^n \to \mathbb{R}^{n+1}$ of the Euclidean space (f an immersion) and the tangent bundle TM of the hypersurface M be an immersed submanifold of the Euclidean space $\mathbb{R}^{2n+2}$. First it's introduced an induced metric on tangent bundle, which we are calling as rescaled induced metric. Second it's defined at the point $(p, u) \in TM$ orthonormal frame of the tangent bundle TM. Then it's investigated some curvature properties of such a tangent bundle by means of orthonormal frame for a given point.

Keywords: Tangent bundle; Hypersurface; Rescaled induced metric; Curvature tensor.

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Screen Generic Lightlike Submanifolds

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Abstract

In this study, we introduce a new class of lightlike submanifolds for indefinite Kähler manifolds which particulary contain invariant lightlike, screen real lightlike and generic lightlike submanifolds and we call this submanifolds as screen generic lightlike submanifolds. After giving an example of a screen generic lightlike submanifold, we investigate the integrability of various distributions and prove a characterization theorem of such lightlike submanifolds in a complex space form. Then, we find necessary conditions for the induced connection to be metric connection.

Keywords: Indefinite Kähler manifold; Lightlike submanifold; Generic lightlike submanifold; Killing horizon.

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Transferring of Subspaces Between Metric Spaces and Comparison of Their Properties

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Abstract

In Euclidean space some subspace are defined using Euclidean metric. Studying the properties of these subspaces, naturally Euclidean metric must be used. When Euclidean space replaced with another metric space, subspaces have important properties in the new space. In this paper, some curves and surfaces are transferred to Lorentz space and also their properties are studied.

Keywords: Euclidean space; Lorentz space; metric space; subspace.

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A New Algorithm to Define the Control Points for a Bezier Curve

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Abstract

The De Casteljau's algorithm gives a Bezier curve using the control points. A matrix can be written different form using Casteljau's algorithm and the coordinate polynomials sorted by power of variable. This matrix gives a new algorithm to find the unknown control points. This paper presents this algorithm.

Keywords: Bezier curve; curve; de Casteljau's algorithm.

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A Study on the One-Parameter Elliptical Planar Motions

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Abstract

One parameter elliptical planar motions have been introduced considering two elliptical planes, of which one is fixed and the other one is moving. Then, the relations between absolute, relative, sliding velocities and accelerations have been obtained. Also, some theorems and results have been given for these velocities and accelerations.

Keywords: One-parameter motions; planar kinematics; elliptical motion.

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On Classification Biharmonic Submanifolds in Complex Projective Space

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Abstract

This study is based on Chen's conjecture which implies that biharmonic submanifolds in space form with non-positive curvature are minimal. But for space forms with positive curvature, results are different. So study of proper biharmonic submanifolds in such a space form is interesting subject. Therefore, we would like to classify proper biharmonic Hopf hypersurfaces in complex projective space with respect to two distinct principal curvatures by using the Hopf map $\pi: S^{2n+1} \to P^n(\mathbb{C})$ and geometric structure of hypersurfaces in S^{2n+1}.

Keywords: proper biharmonic; principal curvature; Hopf hypersurface.

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Bi-Slant Submersions from Kaehler Manifolds

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Abstract

In this presentation, we introduce bi-slant submersions from Kaehler manifolds and give a non-trivial example. The integrability of the distributions and the geometry of the fibers of such submersions are studied. By considering the canonical structures are parallel, some results are obtained. We give some curvature relations between base space and total space.

Keywords: Bi-slant submersion, fiber, distribution, Riemannian submersion.

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Euler-Lagrangian Dynamical Systems with respect to an Almost Product Structure on Tangent Bundle.

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Abstract

The classic mechanics firstly introduced by J. L. Lagrange in 1788. Because of the investigation of tensorial structures on manifolds and extension by using the lifts to the tangent or cotangent bundle, it is possible to generalize to differentiable structures on any space (resp. manifold) to extended spaces (resp. extended manifolds) [4,5,8]. In this study, the Euler-Lagrangian theories, which are mathematical models of mechanical systems are structured on the horizontal and the vertical lifts of an almost product structure in tangent bundle TM. In the end, the geometrical and physical results related to the Euler-Lagrangian dynamical systems are concluded.

Keywords: Euler-Lagrangian equations; Lifts; Almost Product Structure; Tangent Bundle.

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Certain Semisymmetry Curvature Conditions on Paracontact Metric *(K, μ)*-Manifolds

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Abstract

The object of the present paper is to characterize paracontact metric (k,μ)-manifolds satisfying some semisymmetry curvature conditions. We give some basic results of paracontact metric manifolds with characteristic vector field ξ belonging to the (k,μ)-nullity distribution. Also, we study h-projectively semisymmetric and φ-projectively semisymmetric paracontact metric (k,μ)-manifolds. In the last, we show that if a paracontact metric (k,μ)-manifold is Ricci pseudo-symmetric then it is an Einstein manifold.

Keywords: Paracontact metric (k,μ)-manifolds, h-projectively semisymmetry, φ-projectively semisymmetry.

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On Generalization of Pointwise 1-Type Gauss Map

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Abstract

The notion of generalized 1-type Gauss map, which was first introduced in [1], is an extension of 1-type Gauss map and pointwise 1-type Gauss map. In this work, we obtain characterization for surfaces in 3-dimensional Minkowski space with generalized 1-type Gauss map. Then, we study especially surfaces of revolution in 3-dimensional Minkowski space whose Gauss map is of generalized 1-type. Finally, we give the classification theorem of ruled submanifolds in Lorentzian space with generalized 1-type Gauss map.

Keywords: Finite type maps; Generalized 1-type Gauss map, Surface of Revolution; Ruled Submanifolds.

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A Class of Gradient Almost Ricci Solitons

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Abstract

In this study, we provide some classifications for half-conformally flat gradient $f-$almost Ricci solitons (for more details see [1-3]), denoted by (M,g,f), in both Lorentzian and neutral signature. First, we prove that if $||\nabla f||$ is a non-zero constant, then (M,g,f) is locally isometric to a special warped product. On the other hand, if (M,g,f) is isotropic, that is $||\nabla f|| = 0$, then we show that it is locally a Walker manifold. We also construct an example of 4-dimensional steady gradient $f-$almost Ricci solitons in neutral signature.

Keywords: Gradient Ricci (almost) soliton; Half-conformal flatness; Warped product; Walker manifold.

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Relations Between Areas of Lorentz Spherical Regions

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Abstract

In this study, for a one-parameter closed spherical motion $B' = K/K'$, on the 3-dimensional Lorentz space, relationships between the area vector F_X of a closed curve (X) traced on the fixed unit Lorentz sphere K' by a fixed point X chosen from the moving unit Lorentz sphere K, and areas of spherical regions bounded by the spherical orbits on K' of the endpoints of the orthonormal vectors $\{0; \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ chosen from K are given.

Keywords: One-parameter closed Lorentz spherical motion; Lorentz spherical curve; Time-like Steiner vector; Area of a Lorentz spherical region.

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Global invariants of paths in the two-dimensional similarity geometry

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Abstract

Transformations and invariants of curves, surfaces and graphical objects appear in many areas of computer-aided geometric design, computer graphics, computer vision and pattern recognition. Applications of affine, Euclidean and similarity transformations of curves and graphical objects are considered in many works. For example, in the paper [1], a novel and deterministic algorithm is presented to detect whether two given rational plane curves are related by means of a similarity, which is a central question in Pattern Recognition.

For curves in the similarity geometry, using curvatures of the curve in Euclidean geometry, curvature functions of the curve in the similarity geometry were obtained. This method in the similarity geometry give conditions only for local G-similarity of curves, where G is the group of orientation-preserving similarity transformations. (see [2])

Let $E2$ be the 2-dimensional Euclidean space, $Sim(E2)$ be the group of similarities of $E2$ and $Sim^{+}(E2)$ be the group of all orientation-preserving similarities of $E2$.
This presentation concerned with the global invariants of the plane paths under similarity transformations. In this work, the global conditions G- similarity of the plane paths for the groups $G= Sim(E2)$, $Sim^{+}(E2)$ are introduced.

Keywords: Similarity, path, invariant.

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Abstracts of Poster Presentations

A Note on Surfaces of Revolution which Have Lightlike Axes of Revolution in Minkowski Space with Density

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Abstract

In this paper, we study surfaces of revolution in Minkowski space with density. The generating curve of these surfaces satisfies a non-linear second order differential equation which describes the prescribed weighted Gaussian curvature. By solving differential equation we get surfaces of revolution. Also, we give examples of the surface of revolution.

Keywords: Surfaces of revolution; Minkowski space; manifold with density.

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Non-Developable Ruled Surfaces with Density

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Abstract

The aim of this study is to examine the geometry of density ruled surfaces. Offset ruled surfaces with density were defined. The mean and Gaussian curvatures of these surfaces were examined. Then, the relations between the mean curvature of the offset and offset surfaces with density and mean curvature with density and the Gaussian curvature and the Gaussian curvature with density were made.

Keywords: Offset ruled surface, surface with density, offset ruled surface with density

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Smarandache Curves by Harmonic Curvature in Lie Groups

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Abstract

In this study, we introduce special Smarandache curves and obtain Frenet apparatus of a Smarandache curve by harmonic curvature function of a curve in three dimensional Lie groups with a bi-invariant metric. Also, we examine relations between a helix or a slant helix curve and its Smarandache curve in three dimensional Lie Groups.

Keywords: Smarandache curves; Lie groups; helices.

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Spinor Formulation of Involute-Evolute Curves

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Abstract

In this paper, we have studied on spinors with two complex components and we have given spinor representations of Involute Evolute curves in three dimensional Euclidean space. Firstly, we have introduced spinor representations of Frenet vectors of curve in three dimensional Euclidean space. Moreover we have chosen arbitrary two curves which correspond two spinor with complex components. Then, we have considered that these curves are Involute Evolute curves. So, we have investigated the answer of question "How are the relations between the spinors corresponding to the Involute Evolute curves. Finally, we have given an example which crosscheck to theorems throughout this study.

Keywords: Spinors, Involute-Evolute Curves.

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